***Introduction to Theoretical Ecology Assignment 6***

Graphical Analysis of Lotka-Volterra Competition Model

The Lotka-Volterra competition model can be written in terms of the carrying capacities of the two competing species *N1* and *N*2:

, where *r1* and *r2* are the intrinsic population growth rates; *K1* and *K2* are the carrying capacities; *α* is the effect of *N1* on the population growth of *N*2; *β* is the effect of *N2* on the population growth of *N*1.

1. Find all possible equilibrium population sizes of the two species. (5 pts)

***Solution:***

1. Use graphical analysis to determine the stability of the system for all possible scenarios. Please (1) show the stability criteria in each scenario; (2) mark the equilibrium points (both stable and unstable) in the phase plane; and (3) denote all the intercepts between the isoclines and axes. (10 pts)

***Solution:***

Scenario 1: *N1* wins

* Stable equilibrium point: (*K1*, 0)
* Stability criteria: *K1 > K2/β* and *K1/α > K2*

*K1/α*

*K2*

*N2*

*N1 Isocline*

*N2 Isocline*

*K1*

*K2/β*

*N1*

Scenario 2: *N2* wins

* Stable equilibrium point: (0, *K2*)
* Stability criteria: *K2 > K1/α* and *K2/β > K1*

*K2*

*N2*

*N2 Isocline*

*K1/α*

*N1 Isocline*

*K2/β*

*K1*

*N1*

Scenario 3: Stable coexistence

* Stable equilibrium point:
* Stability criteria: *K1/α > K2* and *K2/β > K1*

*K1/α*

*N1 Isocline*

*N2*

*K2*

*N2 Isocline*

*K2/β*

*K1*

*N1*

Scenario 4: Unstable coexistence (saddle)

* Stable equilibrium point: (*K1*, 0) or (0, *K2*)(depending on the initial conditions)
* Stability criteria: *K1* > *K2/β* and *K2 >K1/α*

*K2*

*N2 Isocline*

*N2*

*K1/α*

*N1 Isocline*

*K2/β*

*K1*

*N1*